

CROSS SECTION PROPERTIES

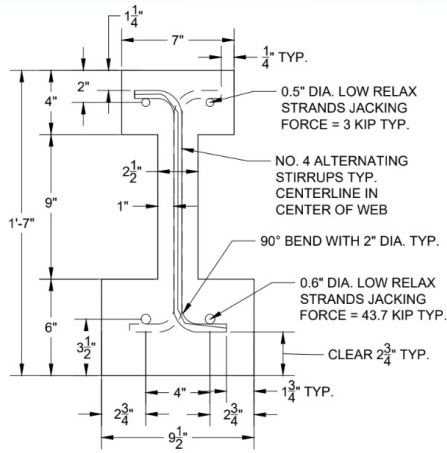


Figure 1: cross section design from shop drawings

$$\begin{aligned}
 L_{tot} &:= 19 \text{ ft} && \text{beam length} \\
 L &:= 18 \text{ ft} && x := 0 \text{ ft}, 0.1 \text{ ft} \dots L \text{ span length} \\
 t_{tf} &:= 3.5 \text{ in} && \text{thickness of top flange} \\
 t_{bf} &:= 6 \text{ in} && \text{thickness of bottom flange} \\
 h_w &:= 9 \text{ in} && \text{height of web} \\
 b_{tf} &:= 9 \text{ in} && \text{width of top flange} \\
 b_{bf} &:= 9.5 \text{ in} && \text{width of bottom flange} \\
 b_w &:= 2.5 \text{ in} && \text{width of web} \\
 h &:= t_{tf} + t_{bf} + h_w = 18.5 \text{ in} && \text{beam height} \\
 L_{p.xs} &:= 2 \cdot (t_{tf} + t_{bf} + h_w) + b_{tf} + b_{bf} + (b_{tf} - b_w) + (b_{bf} - b_w) = 69 \text{ in} && \text{perimeter of cross section}
 \end{aligned}$$

LOADING

$$P_{tot} := 34.9 \text{ kip}$$

total applied actuator load

Predictions

Cracking Load: 22.8 kips
Breaking Load: 34.9 kips

$$P := 0.5 P_{tot} = 17.5 \text{ kip}$$

half of actuator load to each point load

Test Results

Cracking Load: 22.7 kips
Breaking Load: 38.6 kips

MATERIAL PROPERTIES

$$\rho := 124.1 \text{ pcf}$$

$$E_{ps} := 28500 \text{ ksi}$$

$$E_s := 29000 \text{ ksi}$$

$$f_{pu} := 270 \text{ ksi}$$

concrete density (normalweight)

prestressing strand modulus of elasticity

steel reinforcement modulus of elasticity

strength of prestressing strands

Concrete Properties

$$f'_{ci.req} := 5000 \text{ psi}$$

$$f'_{ci} := 5077.5 \text{ psi}$$

$$f'_{c.req} := 8000 \text{ psi}$$

$$f'_c := 7260 \text{ psi}$$

$$d_{agr} := 0.5 \text{ in}$$

Section updated based on cylinder test data

required strength of concrete to cut strands

initial concrete strength (when strands cut)

required strength of concrete to test

strength of concrete (based on ASTM C78)

diameter of aggregate

$$E_{ci} := 33 \cdot (\rho \div \text{pcf})^{1.5} \cdot \sqrt{f'_{ci} \cdot \text{psi}} = 3251 \text{ ksi}$$

initial concrete modulus of elasticity

$$E_c := 33 \cdot (\rho \div \text{pcf})^{1.5} \cdot \sqrt{f'_c \cdot \text{psi}} = 3887 \text{ ksi}$$

concrete modulus of elasticity

$$\lambda := 0.75$$

$$RH := 75$$

Lightweight factor = 1 normalweight, 0.75 lightweight

relative humidity, (%)

STRAND & REINFORCEMENT

Prestressing Strands (7-wire) (bottom)

$$d_{ps} := 0.6 \text{ in}$$

$$A_{ps1} := 0.217 \text{ in}^2$$

$$y_{ps1} := 3.5 \text{ in}$$

$$y_{ps2} := 3.5 \text{ in}$$

$$y_{ps3} := 0 \text{ in}$$

$$n_{ps} := 2$$

$$A_{ps} := A_{ps1} \cdot n_{ps} = 0.434 \text{ in}^2$$

$$w_{ps.ind} := 0.737 \text{ plf}$$

diameter

nominal area

height of prestressing strand 1, from bottom

height of prestressing strand 2, from bottom

height of prestressing strand 3, from bottom

number prestressing strands

total area of prestressing strands

Steel Reinforcement

$$\begin{aligned}d_r &:= 0 \text{ in} \\ A_{r1} &:= 0 \text{ in}^2 \\ y_{r1} &:= 0 \text{ in} \\ y_{r2} &:= 0 \text{ in} \\ n_r &:= 0\end{aligned}$$

diameter
nominal area
height of steel reinforcement strand 1, from bottom
height of steel reinforcement strand 2, from bottom
number of reinforcement bars

$$\begin{aligned}A_r &:= A_{r1} \cdot n_r = 0 \text{ in}^2 \\ w_{r.ind} &:= 0 \text{ plf}\end{aligned}$$

total area of reinforcement
weight of reinforcement bar

$$y_{sbar} := \frac{(A_{ps1} \cdot y_{ps1}) + (A_{ps1} \cdot y_{ps2}) + (A_{ps1} \cdot y_{ps3}) + (A_{r1} \cdot y_{r1}) + (A_{r1} \cdot y_{r2})}{A_{ps} + A_r} = 3.5 \text{ in}$$

centroid of prestressed strands

$$\begin{aligned}d_p &:= h - y_{sbar} = 15 \text{ in} \\ w_{ps} &:= w_{ps.ind} \cdot n_{ps} = 1.474 \text{ plf} \\ w_r &:= w_{r.ind} \cdot n_r = 0 \text{ plf}\end{aligned}$$

height to center of bars
nominal weight of prestressing strands
nominal weight of reinforcing bars

Compression Prestressing strands (top, non functional)

$$\begin{aligned}d'_{ps} &:= 0.5 \text{ in} \\ A'_{ps1} &:= 0.153 \text{ in}^2 \\ n'_{ps} &:= 2 \\ w'_{ps} &:= w_{ps.ind} \cdot n'_{ps} = 1.474 \text{ plf}\end{aligned}$$

diameter
nominal area
number (2 inch vertical spacing)
normal weight

Areas of steel reinforcement and prestressing strands

$$\begin{aligned}A_{ps} &= 0.434 \text{ in}^2 \\ A'_{ps} &:= A'_{ps1} \cdot n'_{ps} = 0.306 \text{ in}^2\end{aligned}$$

area of prestressing strands (bottom)
area of prestressing strands (top)

SHEAR REINFORCEMENT

$$\begin{aligned}no_{legs} &:= 1 \\ A_{stirrup} &:= 0.2 \text{ in}^2 \\ A_v &:= A_{stirrup} \cdot no_{legs} = 0.2 \text{ in}^2 \\ d_A &:= 0.5 \text{ in} \\ s &:= 7 \text{ in} \\ f_y &:= 60 \text{ ksi} \\ L_{stirrup} &:= 19.75 \text{ in} \\ n_{stirrup} &:= 26\end{aligned}$$

number of legs
area of stirrup (No. 4)
shear area of steel
diameter of stirrup (No. 4)
spacing of stirrups
strength of shear reinforcement
length of stirrups
amount of stirrups

$$w_{stirrup} := 0.668 \frac{\text{lb}}{\text{ft}}$$

unit weight of stirrups

Spacing

$$Spacing := \frac{\left(b_{bf} - (d_{ps} \cdot (n_{ps})) - (no_{legs} \cdot d_A) - \left(\frac{5}{8} \text{ in} \cdot 2\right)\right)}{(2)} = 3.3 \text{ in}$$

$$S_{min1} := \max\left(d_{ps}, \frac{5}{8} \text{ in}\right) = 0.63 \text{ in} \quad \text{clear cover (Table 20.5.1.3.3, ACI 318-19)}$$

$$S_{min2} := \max\left(2 \text{ in}, \frac{4}{3} (d_{agr} + d_A)\right) = 2 \text{ in} \quad \text{minimum spacing (25.2.4, ACI 318-19)}$$

Note: for constructability, spacing 2 in. increments center to center

$$check := \text{if}(Spacing \geq \max(S_{min1}, S_{min2}), \text{"OK"}, \text{"Does Not Meet"}) = \text{"OK"}$$

SECTION PROPERTIES - CENTROIDS & INERTIA

Top Flange

$$A_{tf} := t_{tf} \cdot b_{tf} = 31.5 \text{ in}^2$$

area of top flange

Web

$$y_{tf} := \frac{t_{tf}}{2} + h_w + t_{bf} = 16.75 \text{ in} \quad \text{centroid of top flange}$$

$$A_w := b_w \cdot h_w = 22.5 \text{ in}^2 \quad \text{area of web}$$

$$y_w := \frac{h_w}{2} + t_{bf} = 10.5 \text{ in} \quad \text{centroid of web}$$

Bottom Flange

$$A_{bf} := t_{bf} \cdot b_{bf} = 57 \text{ in}^2 \quad \text{area of bottom flange}$$

$$y_{bf} := \frac{t_{bf}}{2} = 3 \text{ in} \quad \text{centroid of bottom flange}$$

Beam

$$y_{barbot} := \frac{(A_{tf} \cdot y_{tf}) + (A_w \cdot y_w) + (A_{bf} \cdot y_{bf})}{A_{tf} + A_w + A_{bf}} = 8.422 \text{ in} \quad \text{centroid of beam, from bottom}$$

$$y_{bartop} := h - y_{barbot} = 10.078 \text{ in} \quad \text{centroid of beam, from top}$$

$$A_g := A_{tf} + A_w + A_{bf} = 111 \text{ in}^2 \quad \text{gross area of concrete}$$

$$V_g := A_g \cdot L = 0.51 \text{ yd}^3 \quad \text{gross volume of concrete, CY}$$

$$e := d_p - y_{bartop} = 4.922 \text{ in} \quad \text{eccentricity}$$

Moment of Inertia

$$I_{tf} := \frac{1}{12} b_{tf} \cdot t_{tf}^3 + A_{tf} \cdot (y_{tf} - y_{barbot})^2 = 2216.701 \text{ in}^4 \quad \text{inertia top flange}$$

$$I_w := \frac{1}{12} b_w \cdot (h - t_{tf} - t_{bf})^3 + A_w \cdot (y_w - y_{barbot})^2 = 249.004 \text{ in}^4 \quad \text{inertia web}$$

$$I_{bf} := \frac{1}{12} b_{bf} \cdot t_{bf}^3 + A_{bf} \cdot (y_{bf} - y_{barbot})^2 = 1846.875 \text{ in}^4 \quad \text{inertia bottom flange}$$

$$I_g := I_{tf} + I_w + I_{bf} = 4312.58 \text{ in}^4 \quad \text{total inertia}$$

$$EI := E_c \cdot I_g = 116416.51 \text{ ft}^2 \cdot \text{kip} \quad \text{EI check}$$

APPLIED LOADS, SHEAR, AND MOMENT

Selfweight

$$w_{swc} := A_g \cdot \rho = 95.7 \text{ plf} \quad \text{concrete selfweight, distributed load}$$

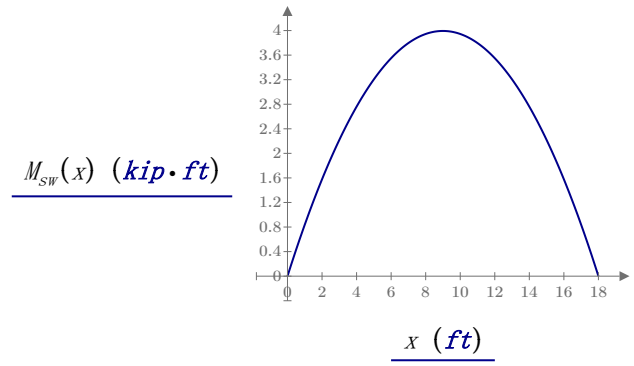
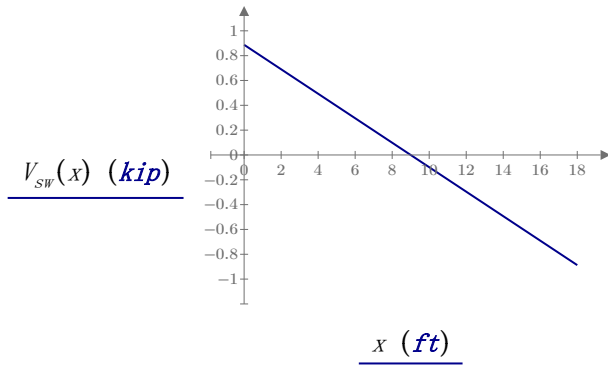
$$w_{sw} := w_{swc} + w'_{ps} + w_{ps} = 98.6 \text{ plf} \quad \text{total self weight, distributed load}$$

$$W_{swt.p} := w_{sw} \cdot L = 1775 \text{ lbf} \quad \text{weight, for shipping \& handling}$$

$$check := \text{if}(W_{swt.p} < 2000 \text{ lbf}, "OK", "Make Lighter") = "OK"$$

$$M_{sw}(x) := \frac{w_{sw} \cdot x}{2} \cdot (L - x) \quad \text{selfweight moment}$$

$$V_{sw}(x) := w_{sw} \cdot (0.5 \cdot L - x) \quad \text{selfweight shear}$$



Live Load

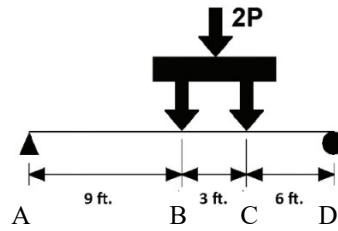


Figure 2: Loading Diagram i

$$AB := 9 \text{ ft} \quad CD := 6 \text{ ft} \quad BC := L - AB - CD = 3 \text{ ft}$$

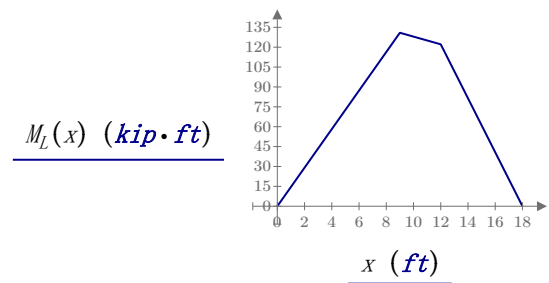
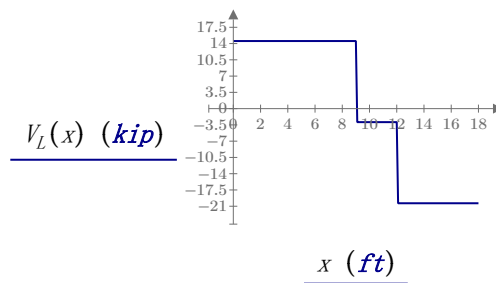
$$L_{ab} := AB = 9 \text{ ft} \quad L_{ac} := AB + BC = 12 \text{ ft} \quad L_{ad} := L = 18 \text{ ft}$$

$$R_A := P \div L \cdot (L - AB + CD) = 14.54 \text{ kip}$$

$$R_D := P \div L \cdot (L - CD + AB) = 20.36 \text{ kip}$$

$$V_L(x) := \text{if}(x \leq L_{ab}, R_A, \text{if}(L_{ab} < x \leq L_{ac}, P \div L \cdot (CD - AB), -R_D))$$

$$M_L(x) := \text{if}(x \leq L_{ab}, R_A \cdot x, \text{if}(L_{ab} < x < L_{ac}, R_A \cdot x - P \cdot (x - AB), R_D \cdot CD - R_D \cdot (x - L_{ac})))$$



Total Loads

$$V_{service}(x) := V_{sw}(x) + V_L(x)$$

total unfactored service shear

$$M_{service}(x) := M_{sw}(x) + M_L(x)$$

total unfactored service moment

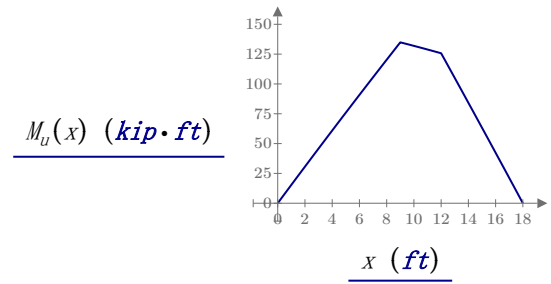
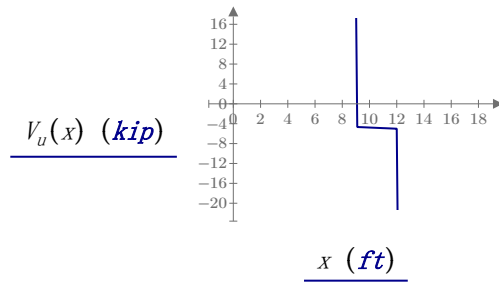
$$V_u(x) := 1.2 \cdot V_{sw}(x) + 1.6 \cdot V_L(x)$$

total factored service shear

(load factors = 1.0 for accurate lab testing)

$$M_u(x) := M_{sw}(x) + M_L(x)$$

total factored service moment



LOSSES

$$f_j := 0.75 \cdot f_{pu} = 202.5 \text{ ksi}$$

jacking stress

Elastic Shortening (ES)

$$P_j := f_j \cdot A_{ps} = 87.9 \text{ kip}$$

jacking force

$$P_{j,perstrand} := \frac{P_j}{n_{ps}} = 43.9 \text{ kip}$$

Jacking force per strand

$$M_{sw,mid} := \frac{w_{sw} \cdot L^2}{8} = 47.924 \text{ kip} \cdot \text{in}$$

max moment of beam (selfweight)

$$k_{ES} := 1.0$$

kes constant - pretentioned members

$$k_{cir} := 0.9$$

keir constant - pretentioned members

$$f_{cir} := k_{cir} \cdot \left(\frac{P_j}{A_g} + \frac{P_j \cdot e^2}{I_g} \right) - \frac{M_{sw,mid} \cdot e}{I_g} = 1.1 \text{ ksi}$$

strand stress at CGS

$$ES := \frac{E_{ps}}{E_{ci}} \cdot k_{ES} \cdot f_{cir} = 9.7 \text{ ksi}$$

elastic shortening

Creep (CR)

$$k_{cr} := \text{if}(\lambda < 1, 1.6, 2) = 1.6$$

creep strain amplifier

$$CR := k_{cr} \cdot \frac{E_{ps}}{E_c} \cdot (f_{cir}) = 12.9 \text{ ksi}$$

creep

Shrinkage (SH)

$$V_{div.s} := A_g \div L_{p.xs} = 1.609 \text{ in}$$

$$k_{sh} := 1.0$$

shrinkage constant

$$SH := \left((8.2 \cdot 10^{-6}) \cdot k_{sh} \cdot E_{ps} \cdot \frac{1}{ft} (1 \text{ ft} - (0.06 \cdot V_{div.s})) (100 - RH) \right) = 5.8 \text{ ksi}$$

Steel relaxation (RE)

$$k_{RE} := 5 \text{ ksi}$$

[PT 5.8.1]

$$J := 0.04$$

[PT 5.8.1]

$$C := 1.0$$

[PT 5.8.2]

$$ju_{ps} := (f_j - ES) \div f_{pu} = 0.714$$

$$C_{ps} := \text{if} \left(ju_{ps} \geq 0.54, \frac{ju_{ps}}{0.21} \cdot \left(\frac{ju_{ps}}{0.9} - 0.55 \right), \frac{ju_{ps}}{4.25} \right) = 0.828 \text{ bottom strands}$$

$$RE := (k_{RE} - J \cdot (SH + CR + ES)) \cdot C_{ps} = 3.2 \text{ ksi}$$

relaxation

STRESS AND FORCE AFTER LOSSES

$$L_{sh} := ES = 9.7 \text{ ksi}$$

short term losses

$$L_{lt} := CR + SH + RE = 21.9 \text{ ksi}$$

long term losses

$$TL := L_{sh} + L_{lt} = 31.6 \text{ ksi}$$

total losses

$$f_{pi} := f_j - ES = 192.8 \text{ ksi}$$

initial stress after ES

$$P_i := A_{ps} \cdot f_{pi} = 83.7 \text{ kip}$$

prestress force after ES at transfer

$$f_{se} := f_j - CR - SH - RE = 180.6 \text{ ksi}$$

final stress after all losses

$$P_e := f_{se} \cdot A_{ps} = 78.4 \text{ kip}$$

prestress force after ES at transfer

FLEXURAL CAPACITY

Code Equations

$$check := \text{if}(f_{se} \geq 0.5 f_{pu}, "OK", "Condition Not Met") = "OK"$$

$$\rho_p := \frac{A_{ps}}{b_{tf} \cdot d_p} = 0.003$$

$$\gamma_p := 0.28$$

gamma factor for prestressing

$$\beta_1 := \max\left(0.65, \min\left(0.85, 0.85 - 0.05 \cdot \left(\frac{f'_c - 4000 \text{ psi}}{1000 \text{ psi}}\right)\right)\right) = 0.687$$

beta strength factor

$$f_{ps} := f_{pu} \cdot \left(1 - \frac{\gamma_p}{\beta_1} \cdot \left(\rho_p \cdot \frac{f_{pu}}{f'_c}\right)\right) = 256.843 \text{ ksi}$$

flexural strength

$$a := \frac{A_{ps} \cdot f_{ps}}{0.85 \cdot f'_c \cdot b_{tf}} = 2.007 \text{ in}$$

depth of Whitney stress block

$$c := \frac{a}{\beta_1} = 2.921 \text{ in}$$

depth to neutral axis

$$c := 3.031 \text{ in}$$

refined guess at c

Refine with Strain Compatibility

$$\epsilon_1 := \frac{f_{se}}{E_{ps}} = 6.3 \cdot 10^{-3}$$

strain due to Pe alone

$$\epsilon_2 := \frac{P_e}{A_g \cdot E_c} \cdot \left(1 + \frac{e^2 \cdot A_g}{I_g}\right) = 2.9 \cdot 10^{-4}$$

increase in steel strain to
decompress concrete at the steel

$$\epsilon_{cu} := 0.003$$

ultimate strain in concrete

$$\epsilon_3 := \epsilon_{cu} \cdot \left(\frac{d_p - c}{c}\right) = 1.2 \cdot 10^{-2}$$

steel strain to failure, concrete crushing

$$\epsilon_{ps} := \epsilon_1 + \epsilon_2 + \epsilon_3 = 1.8 \cdot 10^{-2}$$

total steel strain at failure

$$f_{ps} := \text{if} \left(\epsilon_{ps} \leq 0.0085, 28800 \text{ ksi} \cdot \epsilon_{ps}, 270 \text{ ksi} - \frac{0.04}{\epsilon_{ps} - 0.007} \cdot \text{ksi} \right) = 266.5 \text{ ksi}$$

$$c := \frac{A_{ps} \cdot f_{ps}}{0.85 \cdot f'_c \cdot \beta_1 \cdot b_{tf}} = 3.031 \text{ in} \quad \text{depth of neutral axis}$$

$$a := \frac{A_{ps} \cdot f_{ps}}{0.85 \cdot f'_c \cdot b_{tf}} = 2.083 \text{ in}$$

$$\text{check} := \text{if} (a < t_{tf}, \text{"OK"}, \text{"Condition Not Met"}) = \text{"OK"}$$

$$M_n := \text{if} \left(a > t_{tf}, \left(\left(0.85 \cdot f'_c \cdot t_{tf} \cdot (b_{bf} - b_w) \cdot \left(\frac{a}{2} - \frac{t_{tf}}{2} \right) \right) + A_{ps} \cdot f_{ps} \cdot \left(d_p - \frac{a}{2} \right) \right), A_{ps} \cdot f_{ps} \cdot \left(d_p - \frac{a}{2} \right) \right)$$

$$M_n = 134.5 \text{ kip} \cdot \text{ft} \quad \text{nominal moment}$$

$$\phi_{flexure} := 1.0$$

use 1.0 to calculate lab capacity

$$\phi M_n := \phi_{flexure} \cdot M_n = 134.5 \text{ kip} \cdot \text{ft} \quad \text{factored nominal moment}$$

TRANSFER AND DEVELOPMENT LENGTH

$$l_t := f_{se} \div (3 \text{ ksi}) \cdot d_{ps} = 36.1 \text{ in}$$

$$l_t = 3.01 \text{ ft}$$

transfer length

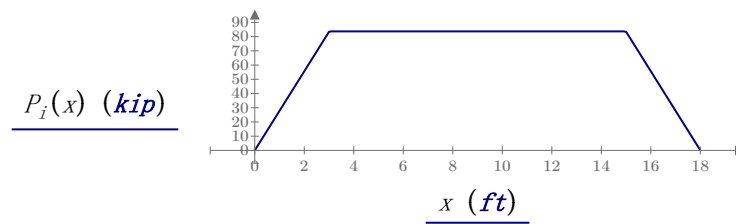
$$l_d := l_t + (f_{ps} - f_{se}) \div (1 \text{ ksi}) \cdot d_{ps} = 87.7 \text{ in}$$

$$l_d = 7.31 \text{ ft}$$

development length

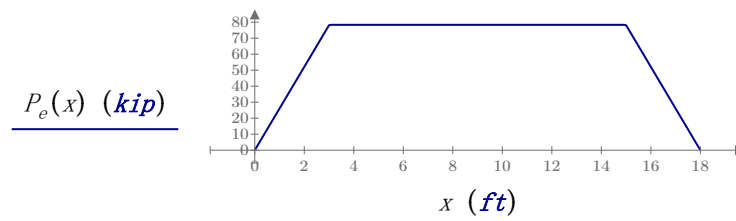
Force at Transfer

$$P_i(x) := \text{if} \left(x \leq l_t, \frac{x}{l_t} \cdot P_i, \text{if} \left(l_t < x \leq L - l_t, P_i, P_i - \frac{P_i}{l_t} \cdot (x - (L - l_t)) \right) \right)$$



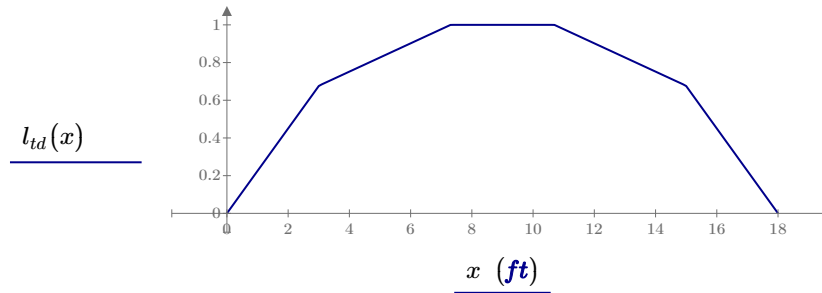
Force at Service

$$P_e(x) := \text{if} \left(x \leq l_t, \frac{x}{l_t} \cdot P_e, \text{if} \left(l_t < x \leq L - l_t, P_e, P_e - \frac{P_e}{l_t} \cdot (x - (L - l_t)) \right) \right)$$



Transfer & Development Length Factor

$$l_{td}(x) := \text{if} \left(x < l_t, \frac{x}{l_t} \cdot \frac{f_{se}}{f_{ps}}, \text{if} \left(l_t \leq x \leq l_d, \frac{x - l_t}{l_d - l_t} \cdot \left(1 - \frac{f_{se}}{f_{ps}} \right) + \frac{f_{se}}{f_{ps}}, \text{if} \left(l_d < x \leq L - l_d, 1, \text{if} \left(L - l_d < x \leq L - l_t, 1 - \frac{x - (L - l_d)}{l_d - l_t} \cdot \left(1 - \frac{f_{se}}{f_{ps}} \right), \frac{f_{se}}{f_{ps}} - \frac{x - (L - l_t)}{l_t} \cdot \frac{f_{se}}{f_{ps}} \right) \right) \right) \right)$$



ALLOWABLE STRESS VS. LIMITS

STRESSES AT TRANSFER

Stress Limits

$$\sigma_{c, end} := -0.7 \cdot f'_{ci} = -3554.3 \text{ psi}$$

allowable compressive stress at end

$$\sigma_{c, mid} := -0.6 \cdot f'_{ci} = -3046.5 \text{ psi}$$

allowable compressive stress at middle

$$\sigma_{tc, PCI}(x) := \text{if} \left(x < 0.25 \cdot L, -0.7 \cdot f'_{ci}, \text{if} \left(0.25 \cdot L < x < 0.75 \cdot L, -0.6 \cdot f'_{ci}, -0.7 \cdot f'_{ci} \right) \right)$$

$$\sigma_{t, end} := 6 \cdot \sqrt{f'_{ci}} \cdot \sqrt{\text{psi}} = 428 \text{ psi}$$

allowable tensile stress at end

$$\sigma_{t, mid} := 3 \cdot \sqrt{f'_{ci}} \cdot \sqrt{\text{psi}} = 214 \text{ psi}$$

allowable tensile stress at middle

$$\sigma_{tt, PCI}(x) := \text{if} \left(x < 0.25 \cdot L, 6 \cdot \sqrt{f'_{ci}} \cdot \sqrt{\text{psi}}, \text{if} \left(0.25 \cdot L < x < 0.75 \cdot L, 3 \cdot \sqrt{f'_{ci}} \cdot \sqrt{\text{psi}}, 6 \cdot \sqrt{f'_{ci}} \cdot \sqrt{\text{psi}} \right) \right)$$

Applied Stress

$$\sigma_{tt}(x) := -\frac{P_i(x)}{A_g} + \frac{P_i(x) \cdot e \cdot y_{bartop}}{I_g} - \frac{M_{sw}(x) \cdot y_{bartop}}{I_g}$$

applied stress at top

$$\sigma_{tb}(x) := -\frac{P_i(x)}{A_g} - \frac{P_i(x) \cdot e \cdot y_{barbot}}{I_g} + \frac{M_{sw}(x) \cdot y_{barbot}}{I_g}$$

applied stress at bottom

Stress Checks

$$\sigma_{tt}(7.4 \text{ ft}) = 100.2 \text{ psi}$$

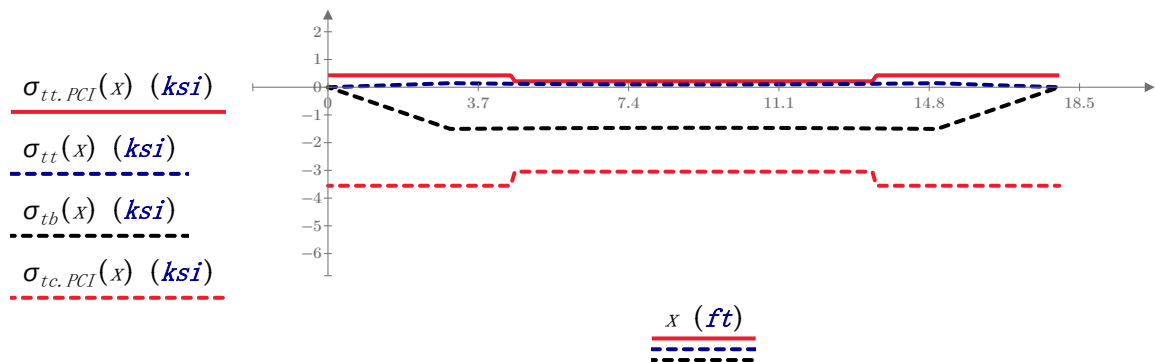
$$\sigma_{tt, PCI}(7.4 \text{ ft}) = 213.8 \text{ psi}$$

$$\sigma_{tt}(11.1 \text{ ft}) = 102.8 \text{ psi}$$

$$\sigma_{tt, PCI}(11.1 \text{ ft}) = 213.8 \text{ psi}$$

$$\sigma_{tb}(7.4 \text{ ft}) = -1467.9 \text{ psi}$$

$$\sigma_{tc, PCI}(7.4 \text{ ft}) = -3046.5 \text{ psi}$$



All stresses due to applied loads are less than the allowable stress limits.

STRESSES AT SERVICE

Stress Limits

$$\sigma_{sc.PCI} := -0.6 \cdot f'_c = -4356 \text{ psi}$$

allowable compressive stress

$$\sigma_{st.PCI} := 7.5 \cdot \sqrt{f'_c} \cdot \sqrt{\text{psi}} = 639 \text{ psi}$$

allowable tensile stress

Applied Stress

$$\sigma_{st}(x) := -\frac{P_e(x)}{A_g} + \frac{P_e(x) \cdot e \cdot y_{barbot}}{I_g} - \frac{M_{sw}(x) \cdot y_{barbot}}{I_g} - \frac{M_L(x) \cdot y_{barbot}}{I_g}$$

applied stress at top

$$\sigma_{sb}(x) := -\frac{P_e(x)}{A_g} - \frac{P_e(x) \cdot e \cdot y_{barbot}}{I_g} + \frac{M_{sw}(x) \cdot y_{barbot}}{I_g} + \frac{M_L(x) \cdot y_{barbot}}{I_g}$$

applied stress at bottom

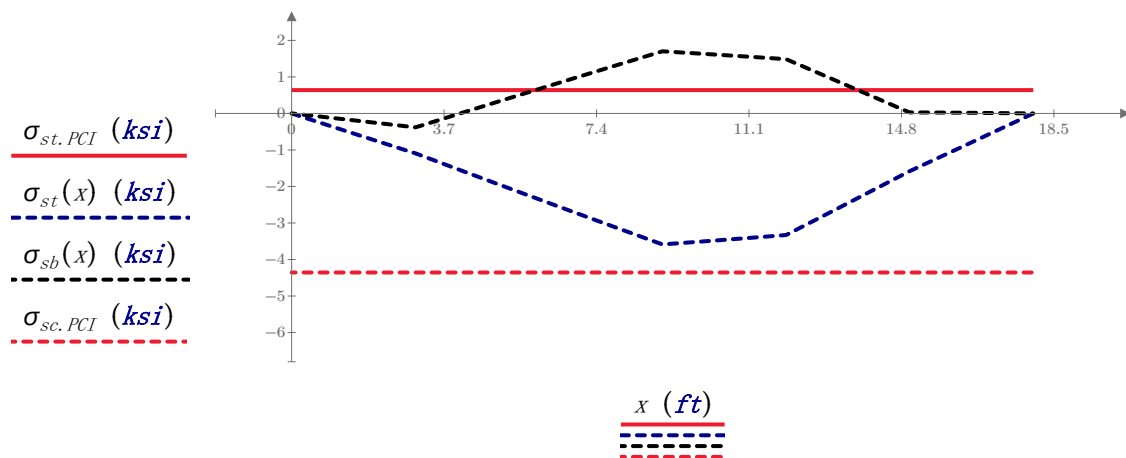
Stress Checks

$$\sigma_{sb}(7.4 \text{ ft}) = 1153.1 \text{ psi}$$

$$\sigma_{sc.PCI} = -4356 \text{ psi}$$

$$\sigma_{sb}(11.1 \text{ ft}) = 1553.1 \text{ psi}$$

$$\sigma_{st.PCI} = 639 \text{ psi}$$



All stresses due to applied loads are not less than the allowable stress limits.

CRACKING MOMENT

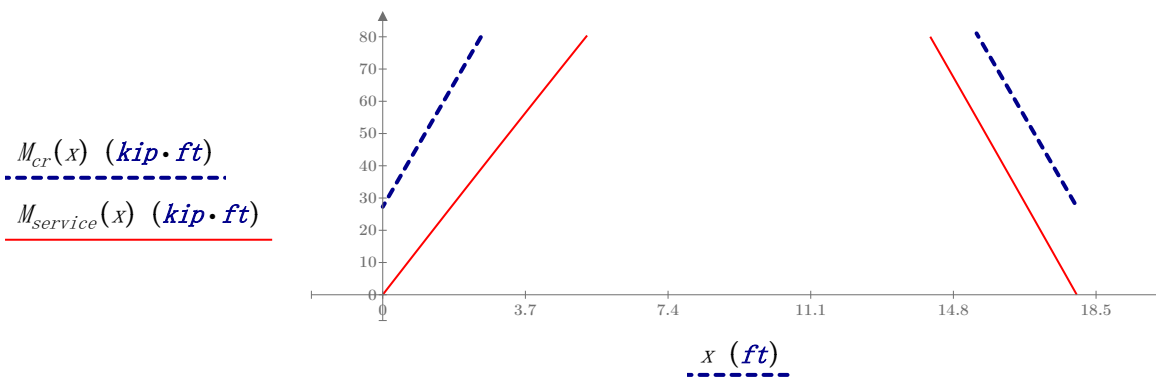
$$f_r := 7.5 \cdot \sqrt{f'_c} \cdot \sqrt{\text{psi}} = 0.639 \text{ ksi}$$

modulus of rupture

Updated based on cylinder
compression results (ASTM C39)

$$M_{cr}(x) := \left(f_r + \frac{P_e(x)}{A_g} + \frac{P_e(x) \cdot e \cdot y_{barbot}}{I_g} \right) \cdot \frac{I_g}{y_{barbot}}$$

cracking moment



$$check := \text{if} (M_{service}(AB) > M_{cr}(AB), "Crack", "Not Crack") = "Crack"$$

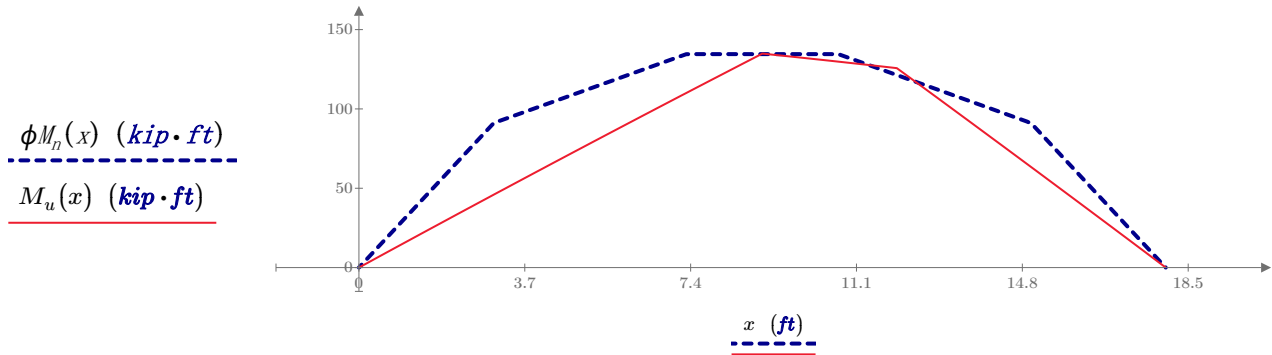
$$M_{service}(AB) = 134.9 \text{ kip} \cdot \text{ft} \quad M_{cr}(AB) = 89.5 \text{ kip} \cdot \text{ft}$$

$$M_{ratio} := M_{service}(AB) \div M_{cr}(AB) = 1.51$$

NOMINAL MOMENT CAPACITY VS. DEMAND

Modified Flexural Capacity

$$\phi M_n(x) := \phi_{flexure} \cdot M_n \cdot I_{td}(x)$$



$$check := \text{if} (M_u(AB) > \phi M_n(AB), "Fail", "Not Fail") = "Fail"$$

$$M_u(AB) = 134.9 \text{ kip} \cdot \text{ft}$$

$$\phi M_n(AB) = 134.5 \text{ kip} \cdot \text{ft}$$

$$M_u(AB) \div \phi M_n(AB) = 1.00$$

SHEAR CAPACITY

$$\phi_{shear} := 1.0$$

resistance factor for shear

$$L_{t, shear} := 50 \cdot d_{ps} = 2.5 \text{ ft}$$

[A22. 5. 7. 1]

Checks to use simple method

$$x_{crit} := \frac{h}{2} = 9.25 \text{ in}$$

$$check := \text{if} (A_{ps} \cdot f_{se} > 0.4 A_{ps} \cdot f_{pu}, "OK to use Simple Method", "Recheck") = "OK to use Simple Method"$$

$$A_v = 0.2 \text{ in}^2$$

area of stirrups

$$V_{cl}(x) := \left(0.6 \cdot \lambda \cdot \sqrt{f'_c \cdot \text{psi}} + 700 \text{ psi} \cdot \min \left(\frac{|V_u(x)| \cdot d_p}{|M_u(x)|}, 1 \right) \right) \cdot b_w \cdot \max(d_p, 0.8 \cdot h)$$

concrete shear strength

$$V_{c, min} := 2 \cdot \lambda \cdot \sqrt{f'_c \cdot \text{psi}} \cdot b_w \cdot d_p = 4.8 \text{ kip}$$

min concrete contribution

$$V_{c, max} := 5 \cdot \lambda \cdot \sqrt{f'_c \cdot \text{psi}} \cdot b_w \cdot d_p = 12 \text{ kip}$$

max concrete contribution

$$V_c(x) := \max(\min(V_{c, max}, V_{cl}(x)), V_{c, min})$$

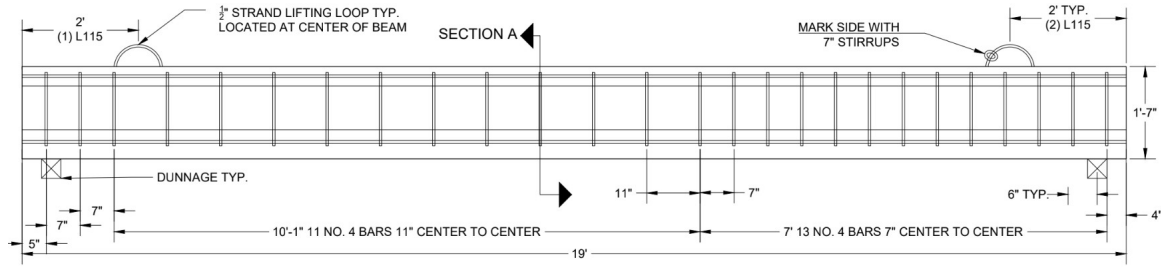


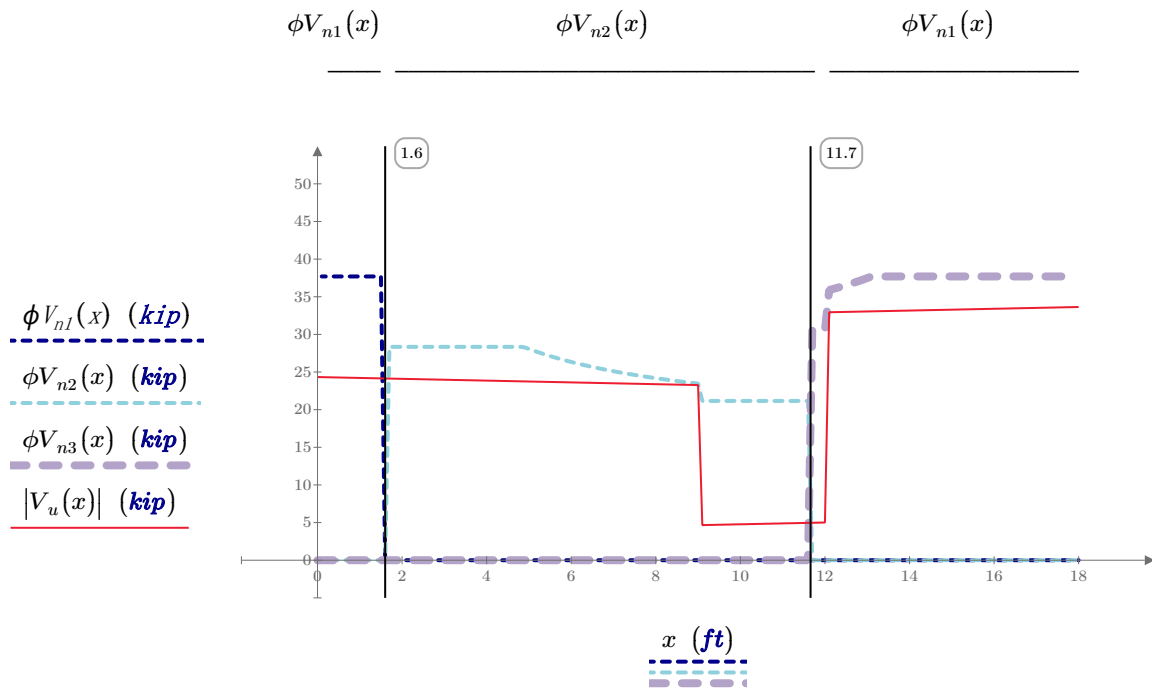
Figure 3: Stirrup Design from shop drawings

s_1	s_2	s_3	spacing
$s_1 := 7 \text{ in}$	$s_2 := 11 \text{ in}$	$s_3 := 7 \text{ in}$	
$V_{s1} := \frac{A_v \cdot f_y \cdot d_p}{s_1} = 25.714 \text{ kip}$	$V_{s2} := \frac{A_v \cdot f_y \cdot d_p}{s_2} = 16.364 \text{ kip}$	$V_{s3} := \frac{A_v \cdot f_y \cdot d_p}{s_3} = 25.714 \text{ kip}$	steel shear strength

$s_{max1} := \text{if} \left(V_{s1} > 4 \cdot \lambda \cdot \sqrt{f'_c \cdot \text{psi}} \cdot b_w \cdot d_p, \min(0.375 \cdot h, 12 \text{ in}), \min(0.75 \cdot h, 24 \text{ in}) \right) = 6.9 \text{ in}$	max stirrup spacing
$s_{max2} := \text{if} \left(V_{s2} > 4 \cdot \lambda \cdot \sqrt{f'_c \cdot \text{psi}} \cdot b_w \cdot d_p, \min(0.375 \cdot h, 12 \text{ in}), \min(0.75 \cdot h, 24 \text{ in}) \right) = 6.9 \text{ in}$	
$s_{max3} := \text{if} \left(V_{s3} > 4 \cdot \lambda \cdot \sqrt{f'_c \cdot \text{psi}} \cdot b_w \cdot d_p, \min(0.375 \cdot h, 12 \text{ in}), \min(0.75 \cdot h, 24 \text{ in}) \right) = 6.9 \text{ in}$	

$A_{v,min} := \min \left(\frac{A_{ps} \cdot f_{pu} \cdot s}{80 \cdot f_y \cdot d_p} \cdot \sqrt{\frac{d_p}{b_w}}, \max \left(0.75 \cdot \sqrt{f'_c \cdot \text{psi}} \cdot \frac{b_w \cdot s}{f_y}, 50 \text{ psi} \cdot \frac{b_w \cdot s}{f_y} \right) \right) = 0.019 \text{ in}^2$	min stirrup area
--	------------------

$\phi V_{n1}(x) := \text{if} (x < 1.6 \text{ ft}, \phi_{shear} \cdot (V_c(x) + V_{s1}), 0)$	Limits for plotting readability
$\phi V_{n2}(x) := \text{if} (1.6 \text{ ft} < x < 11.7 \text{ ft}, \phi_{shear} \cdot (V_c(x) + V_{s2}), 0)$	
$\phi V_{n3}(x) := \text{if} (x > 11.7 \text{ ft}, \phi_{shear} \cdot (V_c(x) + V_{s3}), 0)$	
$(V_u(x))$	factored shear capacity



DEFLECTION BY SUPERPOSITION METHOD

$$\Delta_{cam} := \frac{(A_{ps} \cdot f_{pi}) \cdot L_{tot}^2 \cdot e}{8 \cdot E_{ci} \cdot I_g} = 0.191 \text{ in}$$

deflection due to camber

$$\Delta_{sw} := \frac{-(5 \cdot W_{swt.p} \cdot L^4)}{384 \cdot E_c \cdot I_g} \div 1 \text{ ft} = -0.25 \text{ in}$$

deflection due to selfweight

$$\Delta_{LL,1} := \frac{-P \cdot CD^2 \cdot AB^2}{3 \cdot EI \cdot L} = -0.097 \text{ in}$$

deflection due to point load 6' from end

$$\Delta_{LL,2} := \frac{-P \cdot L^3}{48 \cdot EI} = -0.2 \text{ in}$$

deflection due to point load at midspan

$$check := \text{if} \left(\Delta_{LL,1} + \Delta_{LL,2} < \frac{L}{180}, \text{"Condition Met"}, \text{"Decrease Deflection"} \right) = \text{"Condition Met"}$$

$$\Delta_i := \Delta_{cam} + \Delta_{sw} + \Delta_{LL,1} + \Delta_{LL,2} = -0.375 \text{ in}$$

total deflection

DEFLECTION BY VIRTUAL WORK METHOD

COST

Concrete

$f'_c = 7.26 \text{ ksi}$ strength of concrete, at 28 days

$C_c := \left(20 + \left(\frac{10 \cdot \text{in}^2}{\text{kip}} \cdot f'_c \right) \right) = 92.6$ cost of concrete, \$

Formwork

$L_{p.xs} = 5.8 \text{ ft}$

$C_{fw} := (L_{p.xs} - b_{tf}) \cdot (L) \cdot \frac{1.25}{\text{ft}^2} = 112.5$ cost of formwork, \$

Prestressing Reinforcement

$d_{ps} = 0.6 \text{ in}$ diameter of bottom prestressing strands

$C_{ps} := \frac{0.33}{\text{ft}} \cdot L \cdot (n_{ps}) = 11.9$ cost of bottom prestressing strands, \$

$d'_{ps} = 0.5 \text{ in}$ diameter of top prestressing strands

$C'_{ps} := \frac{0.30}{\text{ft}} \cdot L \cdot (n'_{ps}) = 10.8$ cost of top prestressing strands, \$

Shear Reinforcement

$L_{stirrup} = 19.8 \text{ in}$ length of stirrups

$n_{stirrup} = 26$ amount of stirrups

$w_{stirrup} = 0.7 \frac{\text{lb}}{\text{ft}}$ unit weight of stirrups

$C_{stirrup} := w_{stirrup} \cdot L_{stirrup} \cdot n_{stirrup} \cdot \frac{0.45}{\text{lb}} = 12.9$ cost of stirrups, \$

Total Cost

$C_t := C_c + C_{fw} + C_{ps} + C'_{ps} + C_{stirrup} = 240.6$ total cost, \$

The following unit cost shall be used to determine the beam cost. Concrete cost is based on actual strength, not design strength.

Material	Cost	Notes/Instructions
Concrete Cost (yd³):	\$145/cubic yard (6ksi mix) ≤ \$20 + \$10 (concrete strength ksi) ≤ \$200	Round concrete strength down to nearest ksi
Ultra-High-Performance Concrete	\$400/yd³	
Prestressing Strand:		Use estimated lengths used in the beam
½ in. diameter	\$0.27/ft	
½ in. diameter	\$0.30/ft	
½ in. special	\$0.33/ft	
0.6 in. diameter	\$0.42/ft	
0.7 in. diameter	\$0.55/ft	
Steel:		Use estimated lengths and nominal unit weights in this calculation as provided in the <i>PCI Design Handbook</i>
A615/A706	\$0.45/lb	
Welded Wire (deformed or smooth; for shear)	\$0.60/lb	
Epoxy Coated	\$0.50/lb	
A1035	\$0.70/lb	
Plate Steel	\$0.55/lb	
Forming	\$1.25/ft² of formwork (include all contact surfaces)	

- There is no need to include cost of steel fabrication, concrete fabrication, curing, inserts, etc. Concrete cost is based on actual strength.
- The beam weight shall be estimated by using the measured unit weight of the concrete or by actually weighing the beam. If the beam weight is estimated, it is estimated based on the gross concrete cross section only, ignoring reinforcement, bearing plates, etc. * Special circumstances or special materials not addressed in these rules must be reviewed by the chair of the committee and/or PCI staff.

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